OVERVIEW

The goal of this handout is to present an overview of role of uncertainties in decision analysis and to summarize a few commonly used methods for quantification of uncertainties that may arise in a management problem that uses a simulation model. The handout is divided into the following sections:

1. Decision making under uncertainty
2. Overview of uncertainty quantification methods
3. Method 1: Monte Carlo sampling
4. Method 2: Fuzzy logic

ADDITIONAL READING

These notes present selected and condensed versions of topics in uncertainty estimation methods. Additional readings that will be helpful:

1. Morgan, M.G., Henrion, M. and Small, M., 1992. Uncertainty: a guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge university press.
2. Ross, T.J., 2009. Fuzzy logic with engineering applications. John Wiley & Sons.
3. Lempert, R.J., Popper, S.W. and Bankes, S.C., 2010. Robust decision making: coping with uncertainty. The Futurist, 44(1), p.47.

1. Decision making under uncertainty

Uncertainty is present in all aspects of decision analysis. It is present in the relationships (R) that are used to map levers (L) to measures (M). They are present in the way we implement the levers, i.e., even if we know a suitable course of action for a decision problem, we may not be able to implement it exactly as our computer models suggest. Uncertainties also arise when we try to quantify our preferences in terms of objective functions or measures by which to judge different alternatives. Thus, any formal decision analysis method should be accompanied by quantification of relevant uncertainties and consideration of their impact on the ensuing decision. From the perspective of decision analysis, uncertainties can be classified into two broad categories: *well-characterized* and *deep* uncertainties. These categorizations are relevant as they determine what kind of decision analysis methods will be suitable and what kind of strategies (or levers) the decision analyst should be looking for.

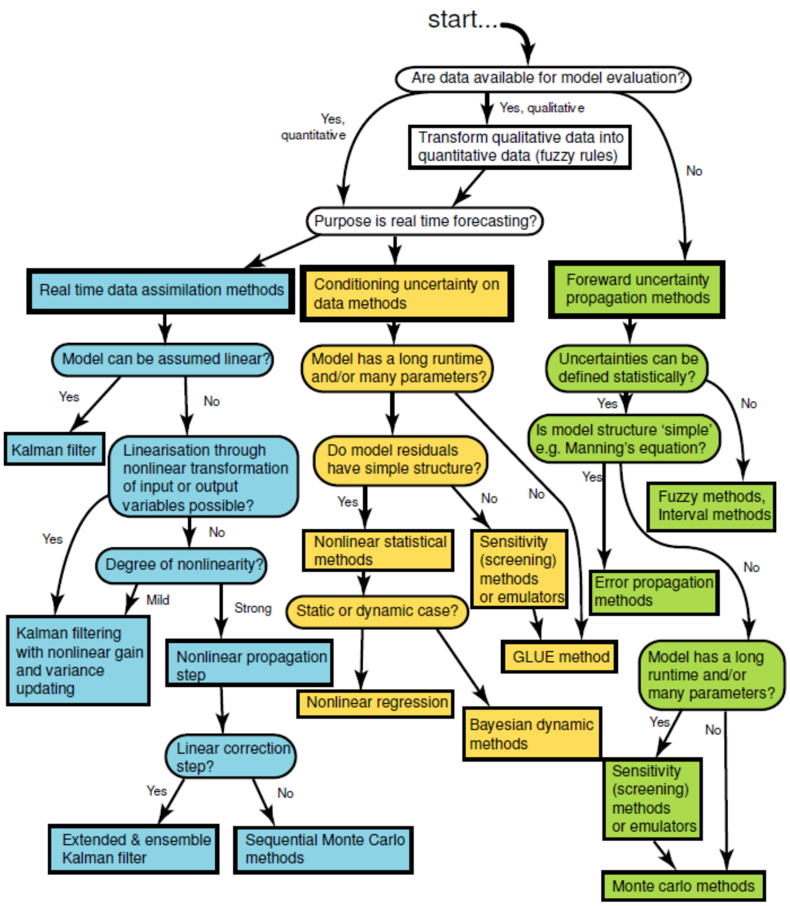
Well-characterized uncertainties are those that can be quantified using standard probability methods. For example, from experiments, one may be able to identify the distribution of hydraulic conductivity of soils in a catchment. The parameters of these distributions can be identified with confidence, even though the exact values of hydraulic conductivity may be uncertain. When a decision problem only has well-characterized uncertainties, one can resort to standard optimization methods (for single or multiple objectives) while accounting for these uncertainties. We can still identify an ‘optimal’ alternative that maximizes the expected value of a measure (in the case of single objective). Let us call this *well-characterized* distribution our *baseline* distribution, i.e., one that is most consistent with our understanding of the system today.

Consider a situation in the future this *baseline* distribution becomes invalid. For example, the future may be such that decision makers cannot agree upon or identify the parameters of the distribution characterizing relevant uncertainties. In such a situation, standard methods of maximizing expected value of objectives (over *baseline* uncertainty) may suggest solutions that degrade severely as soon as the *baseline* assumptions are violated. The possibility of a such as future brings with itself *deep* uncertainties. *Deep* uncertainties are typically associated with long term planning problems, say when we try and think about water availability at the end of century. The socio-economic and environmental uncertainties are such that we cannot simply characterize the them via probability distributions, though we may know the *baseline* distributions of most uncertainties today. When such uncertainties are present, we cannot resort to maximizing expected values of objective functions, as expectations are defined for well-characterized probability distributions! We need new methods that identify strategies that may not be *optimal* (under baseline uncertainties) but will be *robust* under changing assumptions of uncertainty. So, the requirements on the chosen alternative is not just to maximize our objectives but also to avoid possibility of catastrophic failures in some futures. We will learn more about these methods in our decision analysis lectures.

2. Overview of uncertainty quantification methods

Literature is rich with methods for quantification of uncertainties. But this richness also brings a challenge to the uninitiated decision analyst who may stumble upon terms such as: Monte-Carlo, Bayesian analysis, fuzzy logic, interval analysis, possibility theory, and so and so forth. It is quite easy to get lost in the uncertainty of the uncertainty quantification methods! One way to approach the issue of methodological uncertainty is to start with the decision problem and think about what may be needed. For example, if it is a short-term decision problem with a well-defined understanding of uncertain parameters, probability theory may suffice. However, if the planning horizons are long, the system being analyzed is quite complex, and/or the measures are not easily quantified, scenario analysis, fuzzy logic, or possibility theory may be worth exploring. In short, let the problem guide the choice of methods (and not the other way around!). Of course, this will be a bit iterative, as after going through one cycle of decision analysis, the analyst may realize that some uncertainties need alternative representations. There are some flow-charts to guide the analyst on which uncertainty representation method is best for them given availability of data and requirements of the problem (Figure 1). These methodological summaries are quite useful, if available for the domain of interest.

From the vast choice of uncertainty quantification methods, I discuss here two methods in some details as they have emerged to be useful in a wide variety of decision problems. The two methods are also mutually exclusive as they represent different types of uncertainties. So, one may use both at the same time in a decision problem in case the requirement arises. I do not discuss here another commonly used method: ‘scenario’ analysis. Lempert et a. (2010) cited in additional reading provide a nice overview.



From. Beven, K., 2010. Environmental modelling: An uncertain future? CRC Press.

3. <u>Method 1: Monte Carlo sampling</u>

When uncertainties can be represented by probability distributions, Monte Carlo sampling can be used to estimate uncertainty in dependent output variable given uncertainty in (one or more) independent input variables. For simple functions, whose first and higher order derivatives are available, uncertainty can be quantified analytically. If one knows the parameters of the probability distribution characterizing the uncertainty in inputs, one can analytically derive the shape and parameters of probability distribution characterizing the uncertainty in the output. However, often, the functions that map inputs to outputs (or levers to measures for decision analysis) are complex as they aim to represent natural and/or socio-economic systems. For example, a typical hydrologic model may have several parameters and several equations that translate inputs to output. In such cases, it is not possible to analytically derive the parameters and shape of the probability distribution of outputs given the distributions of inputs. Monte-Carlo sampling is used in these cases. The sequence of steps to be followed when using Monte-Carlo sampling to quantify uncertainty in outputs:

1. Identify sources of uncertainties in the model. In the context of hydrologic modeling, let's say 'm' model parameters are uncertain.
2. Characterize these uncertainties using appropriate probability distributions.

3. Now sample from the probability distributions to create random sets of inputs for the mode. You can do this in two ways (depending upon the information available to you):
   a. Assume uncorrelated inputs: in this case you can either use uniform random sampling (URS) or Latin Hypercube Sampling (LHS). In URS, you simply sample each parameter one at a time and then put them together in a matrix. For example, if you create 'N' samples of each parameter, a 'N x m' matrix can be create. Each row of this matrix contains one possible realization of parameters that can be used to run the model. LHS is a type of *stratified* sampling that attempts to provide better coverage of the space of input parameters as compared to URS. If the available online packages for LHS will allow you to create samples only from uniform distribution you need to follow these steps to create distributions of your choice:
      i. Create a 'N x m' matrix of numbers sampled from a uniform distribution using LHS.
      ii. Treat each column (of size N x 1) of this matrix as the cumulative distribution function (CDF) of the corresponding parameter. Invert the CDF values to obtain random samples for that parameter. Suppose that the 1^{st} number for column 1^{st} is Y. If the cumulative distribution function of the 1^{st} parameter follows a distribution 'F', you are looking for $X = F^{-1}(Y)$, where $F(X) = P(x \leq X) = \mathrm{Y}$ is the cumulative distribution function.
      iii. Repeat (ii) for each parameter, you will end up with 'N x m' matrix of parameters that are now sampled from their specified distributions.
   b. If inputs are correlated: in this case you not only need to preserve the marginal distributions of each parameter, but also correlations between them. Note that methods in (a) above do not take care of correlation between parameters, they will generally create parameters uncorrelated with each other. The theory of copulas comes in handy there. We do not cover this in the course but you can extend the concepts from (a i-iii) to introduce correlations between parameters fairly easily.

4. <u>Method 2: Fuzzy logic</u>

Fuzzy logic deals with uncertainties that are different from chance uncertainties in probability theory. If there is any imprecision in quantifying information, fuzzy logic can be used. Imprecision arises from different sources in decision analysis. It can arise because we are trying to quantify some qualitative information. For example, 'remove the cake from the oven when it turns light brown'. Each person may choose their own 'light brown' threshold. It can also arise because we are not sure about some thresholds or cutoffs that differentiate between one type of outcome with another. For example, an instructor may be uncertain whether a student scoring 89.9 should be awarded a grade A (where a grade A is awarded on scores 90 and above)[1]. A hydrologic modeler may be uncertain whether a Nash Sutcliffe Efficiency (NSE) of 0.7 is acceptable or not. Note that NSE ranges from negative infinity (worst value) to one (best value). The basics of fuzzy logic are in set theory. In standard set theory an element

[1]In case you are curious about the rules for students scoring 89.9 in CE766, we will be rounding off the numbers, so anyone above 89.5 gets an A grade.

either belongs to a set or not (membership of 1 or 0). Fuzzy logic generalizes this to include different levels of memberships. Let's take the example of student scores mapped to grades. If a course instructor creates a rule that anyone scoring 90 or above in the class gets A grade, setting a hard deadline at 90, these are *crisp* sets (Figure 1a). However, in the fuzzy world, an instructor can construct a fuzzy membership function for A grade as shown in Figure 1b. So a student scoring 89.5 will have a membership of 0.5 for A- and 0.5 for A. Here we assume that membership for grade A increases linearly from 0 to 1 as we go from 89 to 90. Similarly, membership for grade A- decreases linearly from 1 t 0 as we go from 89 to 90.

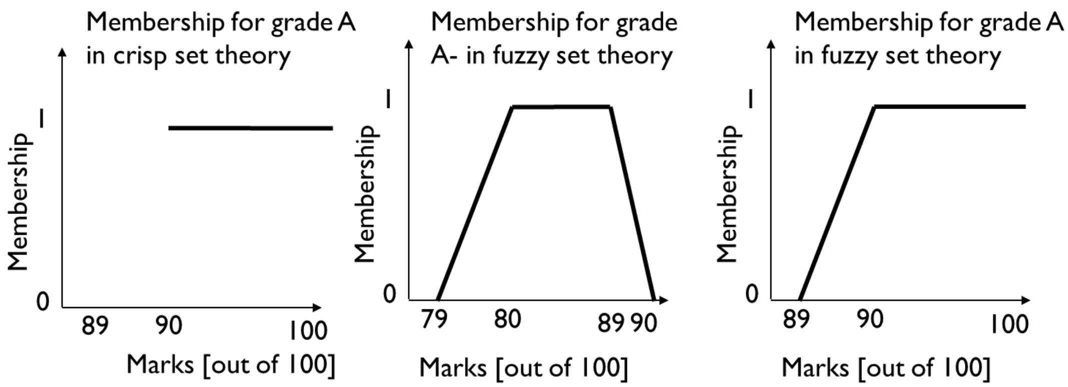


Figure 2. Crips vs. fuzzy membership functions for grade assignments in a course.

We can use the example above to generalize the definition of a membership function (Figure 3). Areas of the function that are <1 but >0 constitute the boundary of the function, while areas with membership of 1 constitute the core. Membership functions come in a variety of shapes (not necessarily linear). They can be used to represent uncertainties related to imprecise information and integrated in the modeling process. Refer to Ross (2009) and lecture slides for further information.

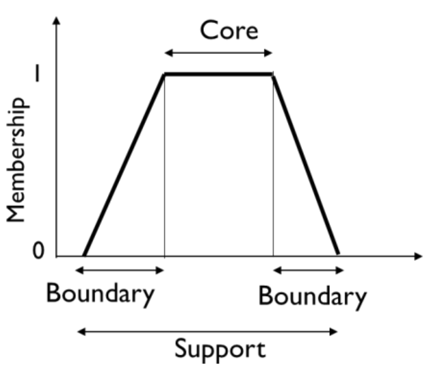


Figure 3. Defining membership functions.