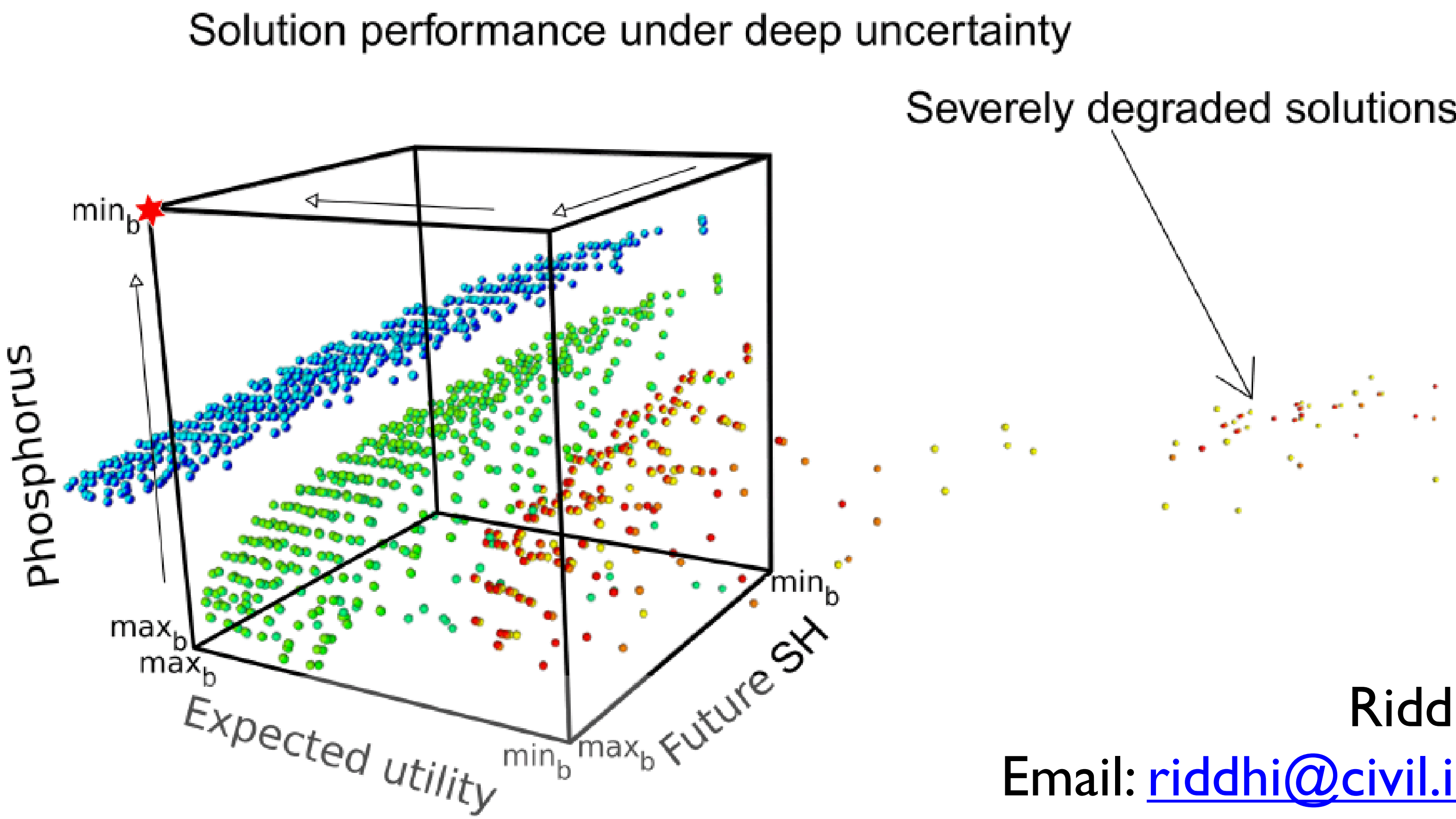
DECISION ANALYSIS-III

CE 766 LECTURE 17



Riddhi Singh
Email: riddhi@civil.iitb.ac.in

Today, we will learn about…

- Tackling uncertainties in optimization: well characterized uncertainties
- Deep uncertainties: robust decision making
- Multi-objective robust decision making

WELL CHARACTERIZED UNCERTAINTIES

Examples of well-characterized uncertainties

- Reservoir optimization: uncertain inflows sampled from chosen distributions
- Hydrology simulations: methods to estimate uncertainty in parameters using observations
- Ecology simulations: probabilistic approaches to model measurement and parameter uncertainties

Stochastic optimization: dealing with well-characterized uncertainties

Stochastic optimization (**SO**) methods are optimization methods that generate and use random variables. For stochastic problems, the random variables appear in the formulation of the optimization problem itself, which involve random objective functions or random constraints. Stochastic optimization methods also include methods with random iterates. Some stochastic optimization methods use random iterates to solve stochastic problems, combining both meanings of stochastic optimization.[1] Stochastic optimization methods generalize deterministic methods for deterministic problems.

https://en.wikipedia.org/wiki/Stochastic_optimization

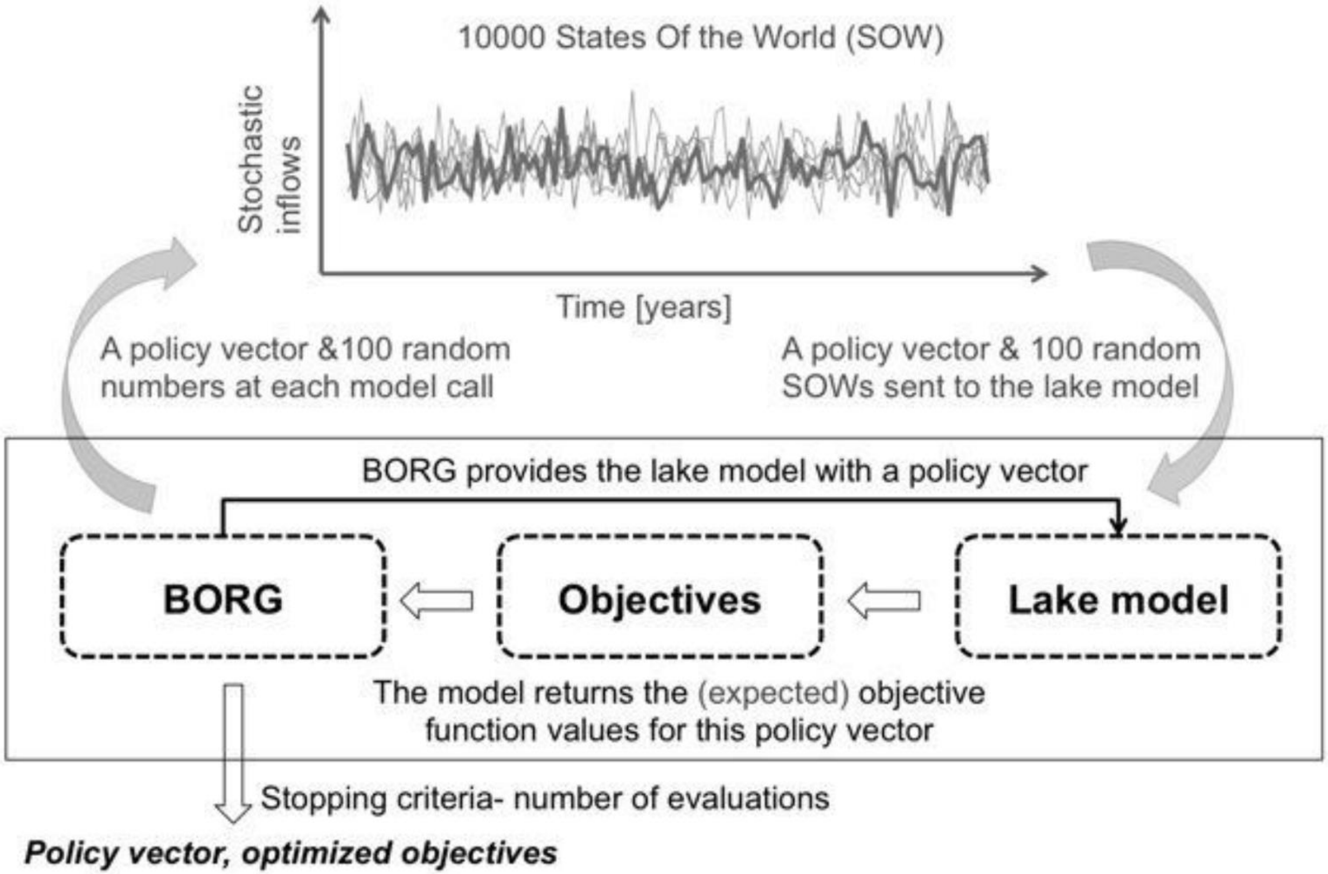
Optimization is stochastic when:

1. Underlying model or data are random
   – Stochastic version of the lake problem
2. The search method itself has some randomness built into it
   – heuristics that employ some type of random search

<u>How to do it?</u>

Replace the deterministic objective function with its stochastic variant

Stochastic optimization when SOWs are too large



Singh, R., Reed, P.M. and Keller, K., 2015. Many-objective robust decision making for managing an ecosystem with a deeply uncertain threshold response. *Ecology and Society*, *20*(3), pp.1-32.

DEEP UNCERTAINTIES: RDM

Deep uncertainties are represented by

- Scenarios: possible states of the world (SOWs) that may happen in the future
- It is not easy to assign probabilities to these SOWs
- Optimization over well characterized uncertainties may be the first step but there should be further testing on strategy performance across SOWs

Robust decision making

- A decision analysis framework for dealing with deep uncertainties in long term policy problems
- Focuses on ‘robustness’ or the ability of a strategy to minimally degrade when conditions change from baseline
  - Single objective robustness
  - Multi-objective robustness

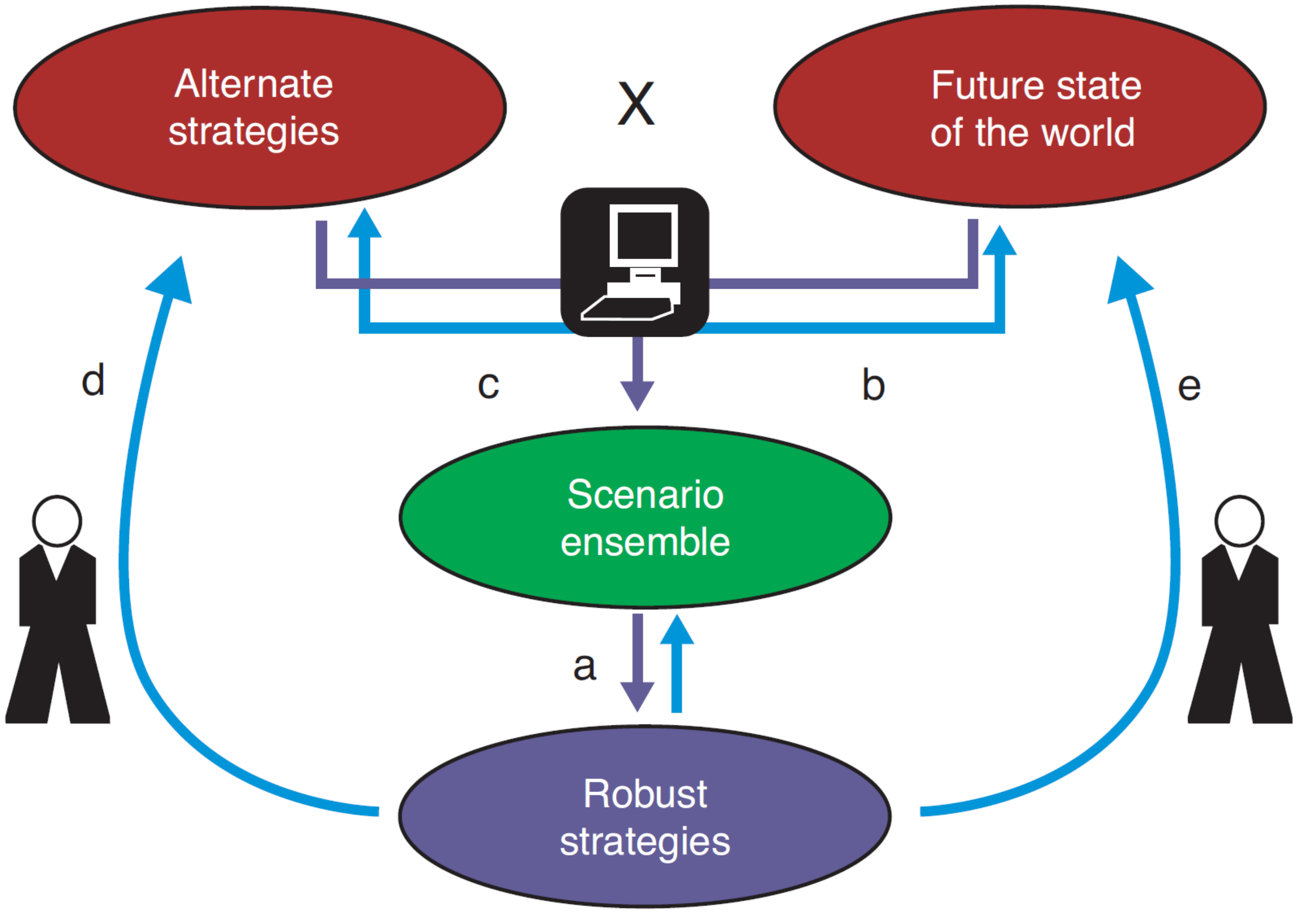
Dealing with deep uncertainty in decision analysis

- Consider large ensembles (hundreds to millions) of scenarios
- Seek robust, not optimal, strategies
- Achieve robustness with adaptivity
- Design analysis for interactive exploration of the multiplicity of plausible futures

Iteration is a key part of decision analysis as decision makers explore various scenarios and gain understanding of consequences of their choices

Lempert, R.J., 2003. *Shaping the next one hundred years: new methods for quantitative, long-term policy analysis*. Rand Corporation.

Interaction of humans with computer simulations is key to RDM



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Robust vs. optimal strategies

- Optimal strategies: those strategies that optimize decision maker's objectives under standard risk
  - No guarantee of performance in a different future
- Robust strategies: focus on performance across a large range of SOWs
  - May be optimal in some scenarios, sub-optimal in others
  - Preferred over near term strategies that are very 'sensitive' to baseline problem setup

Quantifying robustness: regret based approaches

- Regret is defined as the difference between the performance of a future strategy, given some value function, and that of what would have been the best performing strategy in that same future scenario. Thus, following Savage (1950), the regret of strategy j in future f using values m is given by:

$$\mathrm{Regret}_{\mathrm{m}}\left(j,f\right)=\underset{j'}{Max}\left[Performance_m\left(j',f\right)\right]-Performance_m\left(j,f\right)$$

- The difference between strategies that would have been chosen with 'perfect foresight' or knowledge of the future and what was actually chosen

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Alternative regret criteria: relative regret

$$\text{Relative\_Regret}_{\text{m}}\left(j, f \right) = \frac{\underset{j'}{Max}\left[Performance_m\left(j', f \right) \right] - Performance_m\left(j, f \right)}{\underset{j'}{Max}\left[Performance_m\left(j', f \right) \right]}$$

- Relative measures are generally more useful as they present results in a context that can be easily understood (I may be more concerned about deterioration in performance of $100 when the best strategy is $200 instead of $10000

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Robustness can then be defined as:

- Attribute of a strategy that has a small regret or relative regret over a large range of plausible SOWs
- Regret can be calculative using:
  - Single objective
  - Multiple objectives: regret in this case will be multi dimensional, you can do better in one objective and worse in another
- Alternative definitions of robustness exist in literature such as those based on 'satisficing' performance

*note that till now we assume that strategies are enumerated *apriori*

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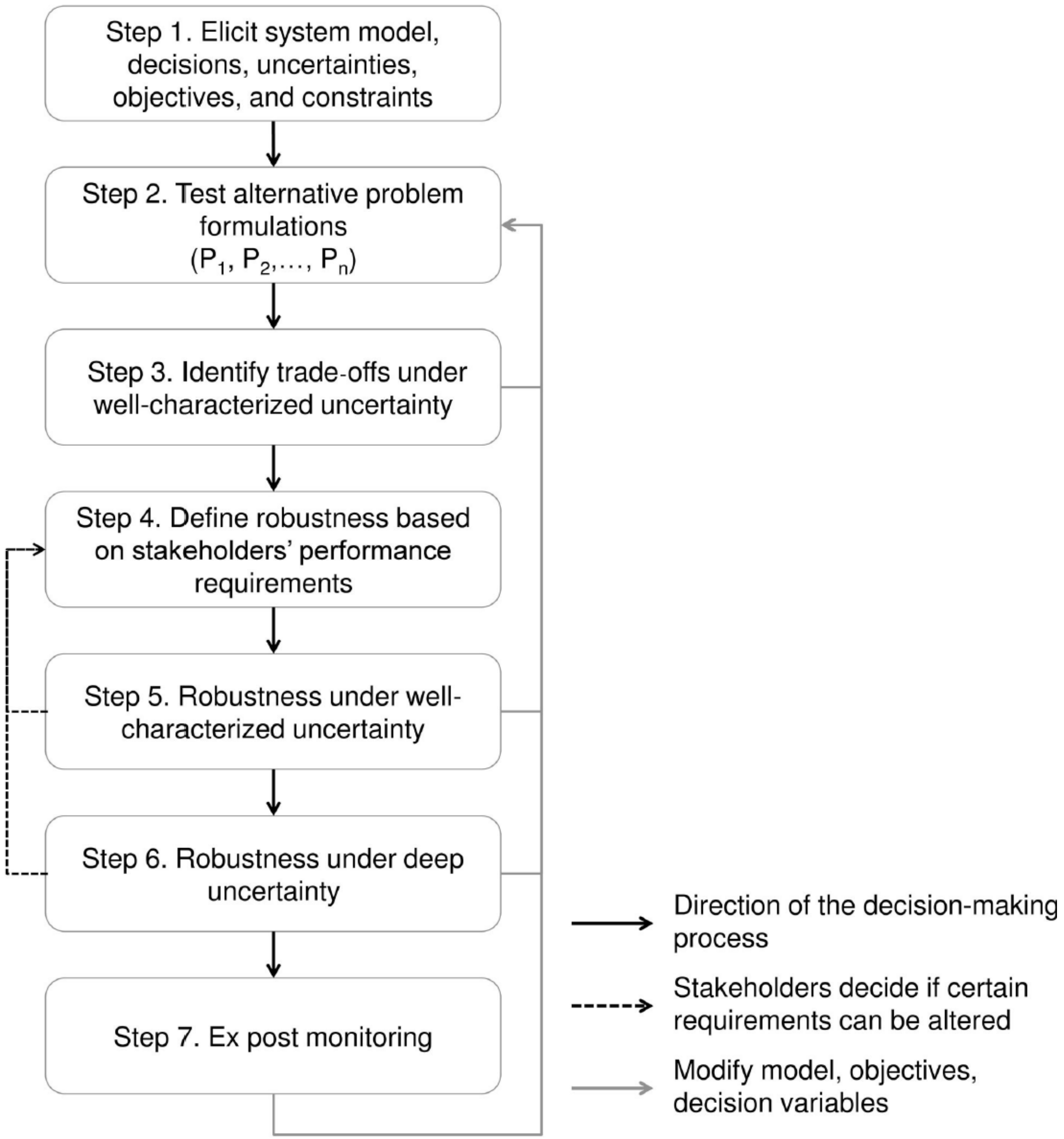
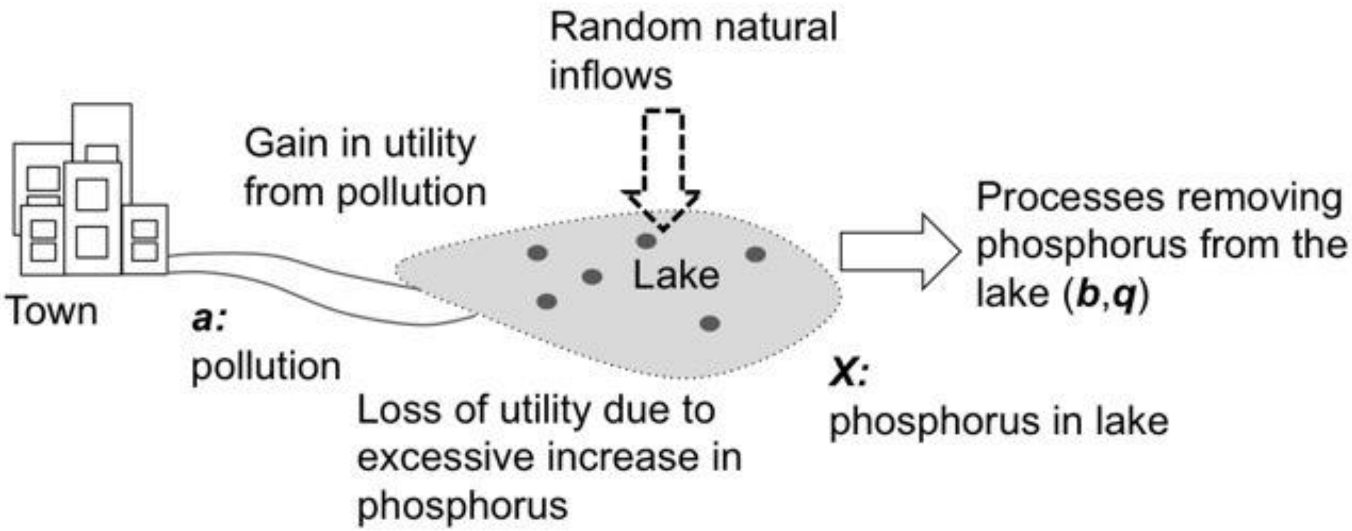
MULTI-OBJECTIVE ROBUST DECISION MAKING

Example from: Singh, R., Reed, P.M. and Keller, K., 2015. Many-objective robust decision making for managing an ecosystem with a deeply uncertain threshold response. *Ecology and Society*, *20*(3), pp.1-32.

Multi-objective robust decision making



MORDM for the lake problem



$$X_{t+1} = X_t + a_t + \frac{X_t^q}{1+X_t^q} - bX_t + \ln(\mu,\sigma)$$

Objectives: (1) discounted net present value of expected utility (maximize), (2) average levels of phosphorus in the lake (minimize), (3) expected utility of the present stakeholder (maximize), (4) expected utility of the future stakeholders (maximize), and (5) reliability of keeping the lake below the eutrophication threshold (maximize).

Problem formulations

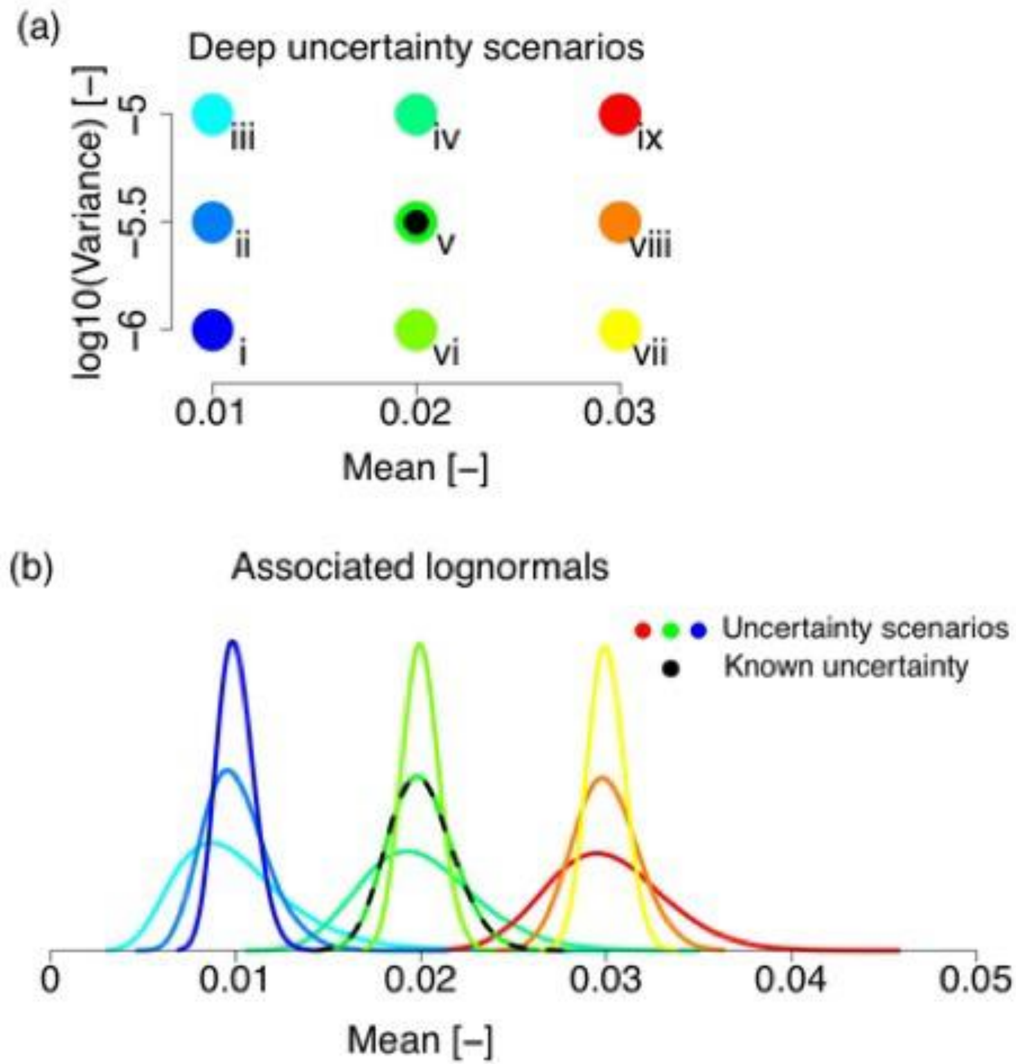
1. Deterministic single objective
2. Stochastic single objective

$$F(x) = O_1\big|_N ,$$
$$\forall x \in \Omega$$
$$x = (a_1, a_2, \ldots, a_{20})$$
$$N = \begin{cases} 1 & for\ P1 \\ 10000 & for\ P2 \end{cases}$$

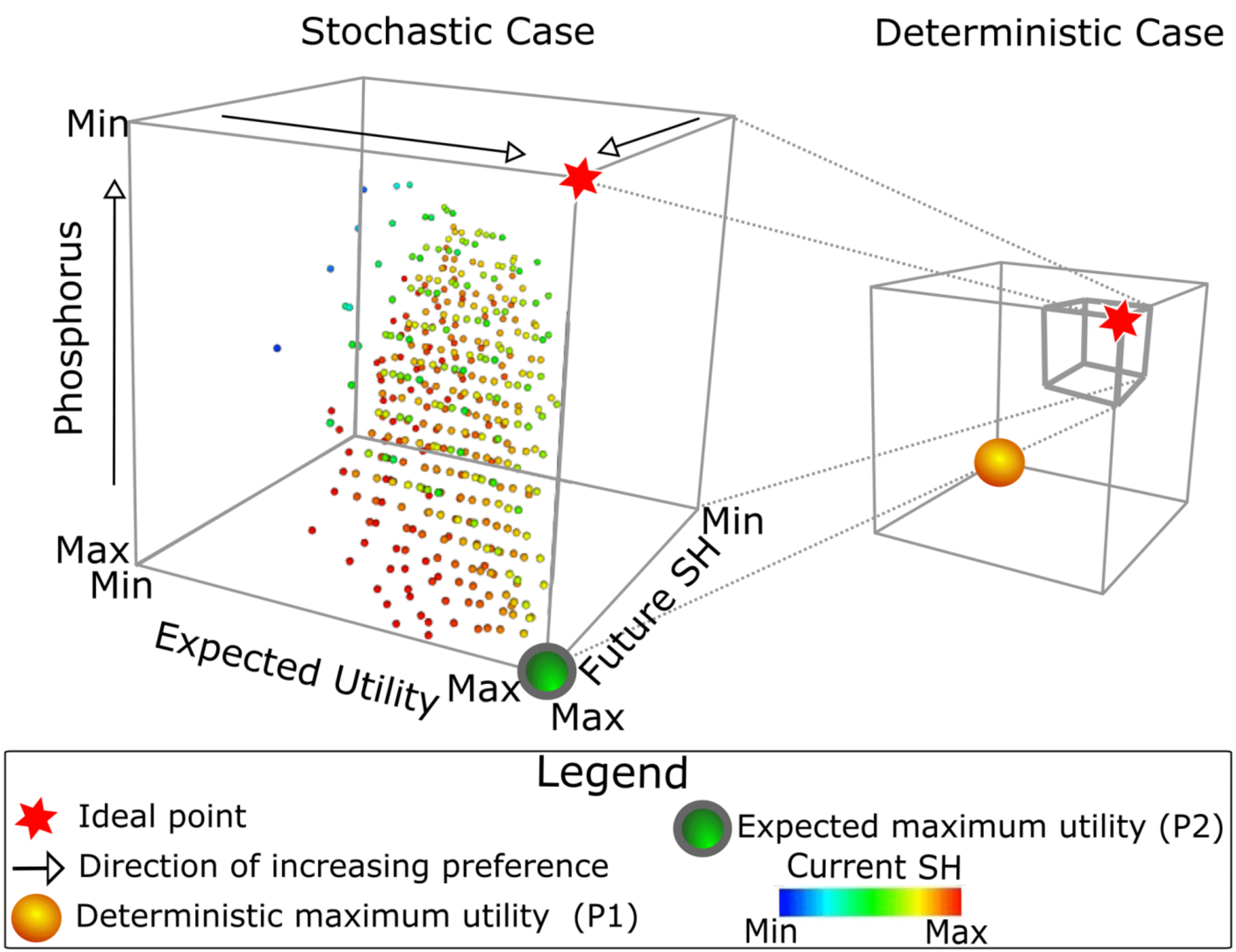
3. Stochastic multi objective

$$F(x) = (O_1, O_2, O_3, O_4, O_5)\big|_N ,$$
$$\forall x \in \Omega$$
$$x = (a_1, a_2, \ldots, a_{20})$$
$$N = 10000$$
$$subject\ to,\ c_{rel} : O_5 > 0.9$$

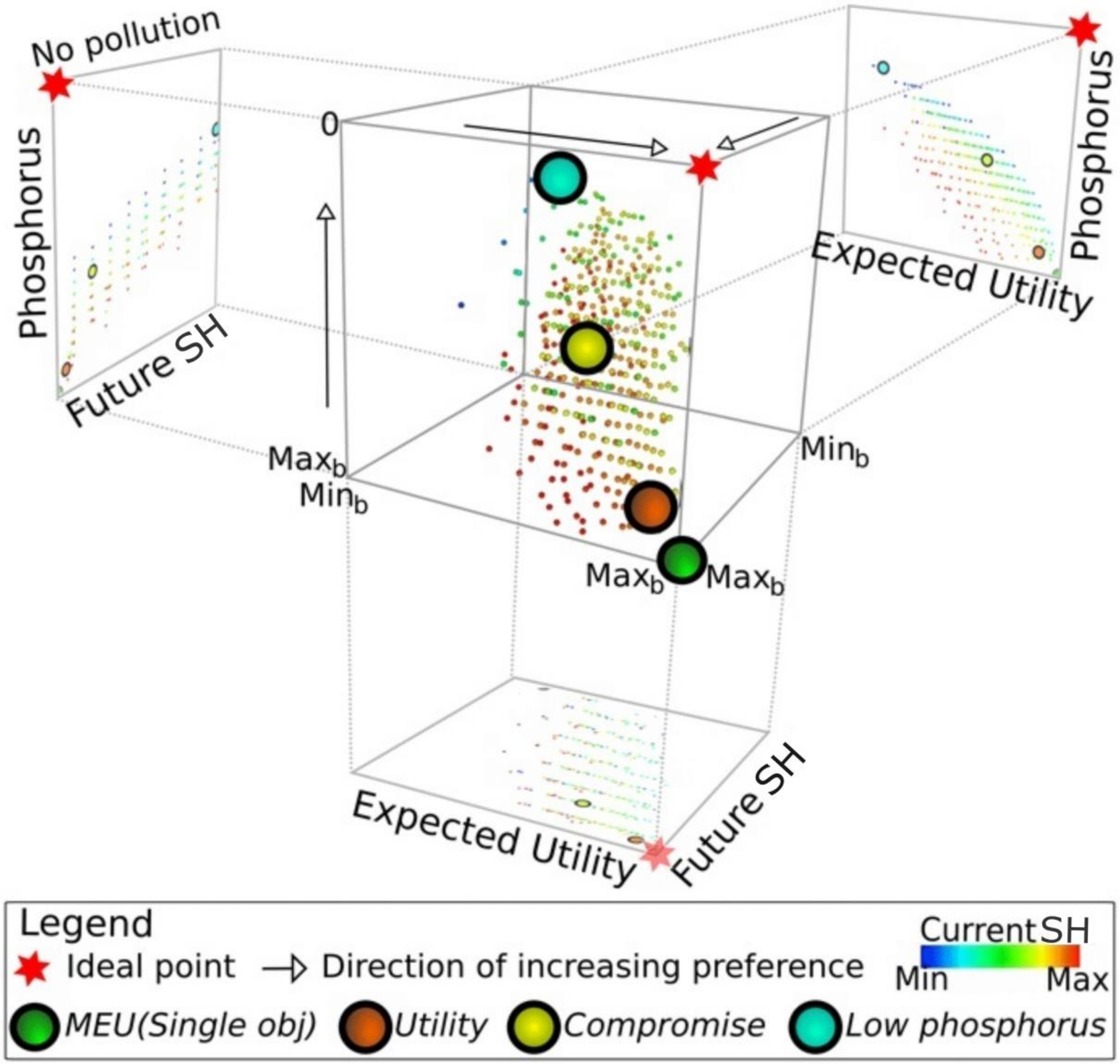
Uncertainty specifications



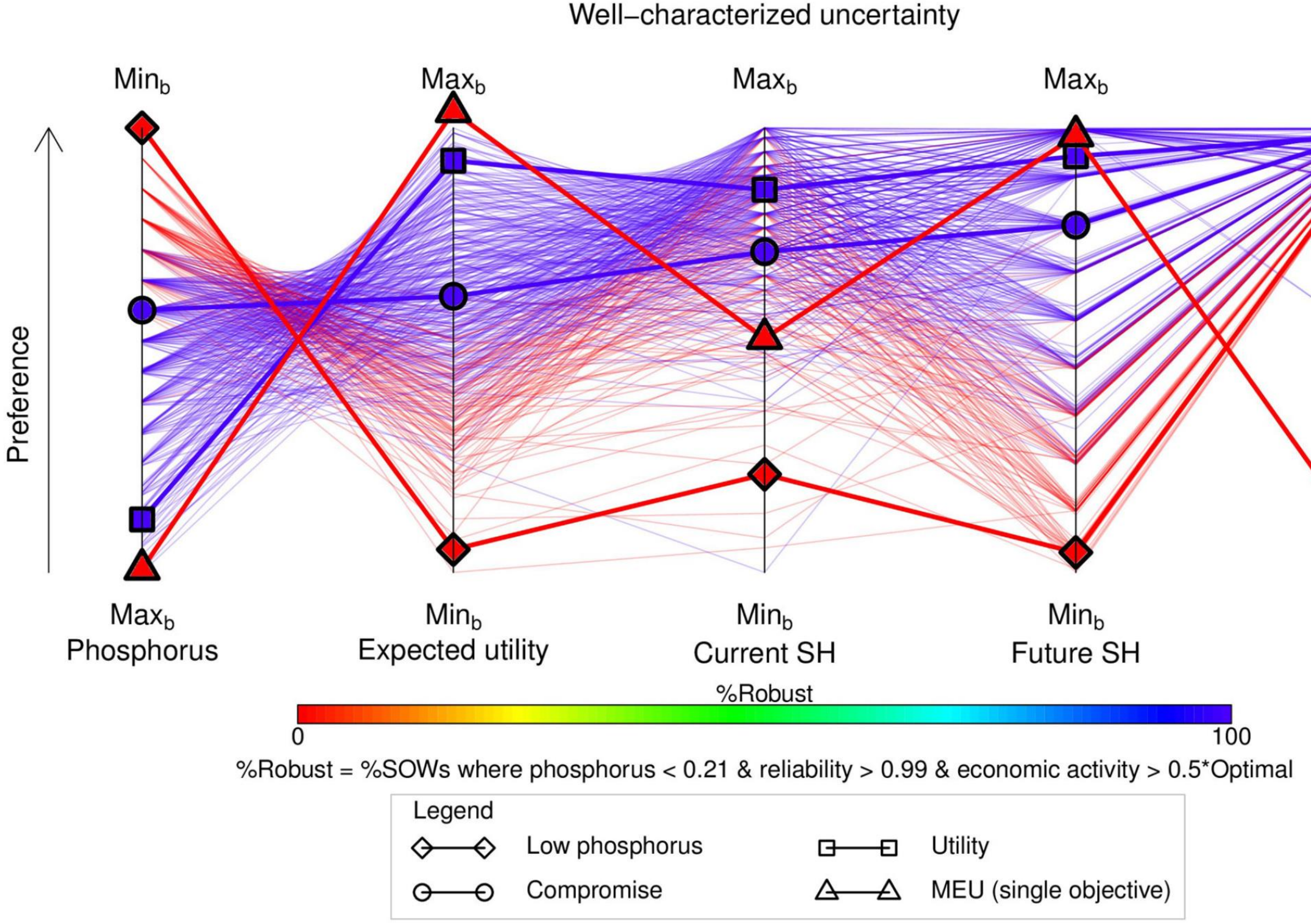
Tradeoffs under well-characterized uncertainties



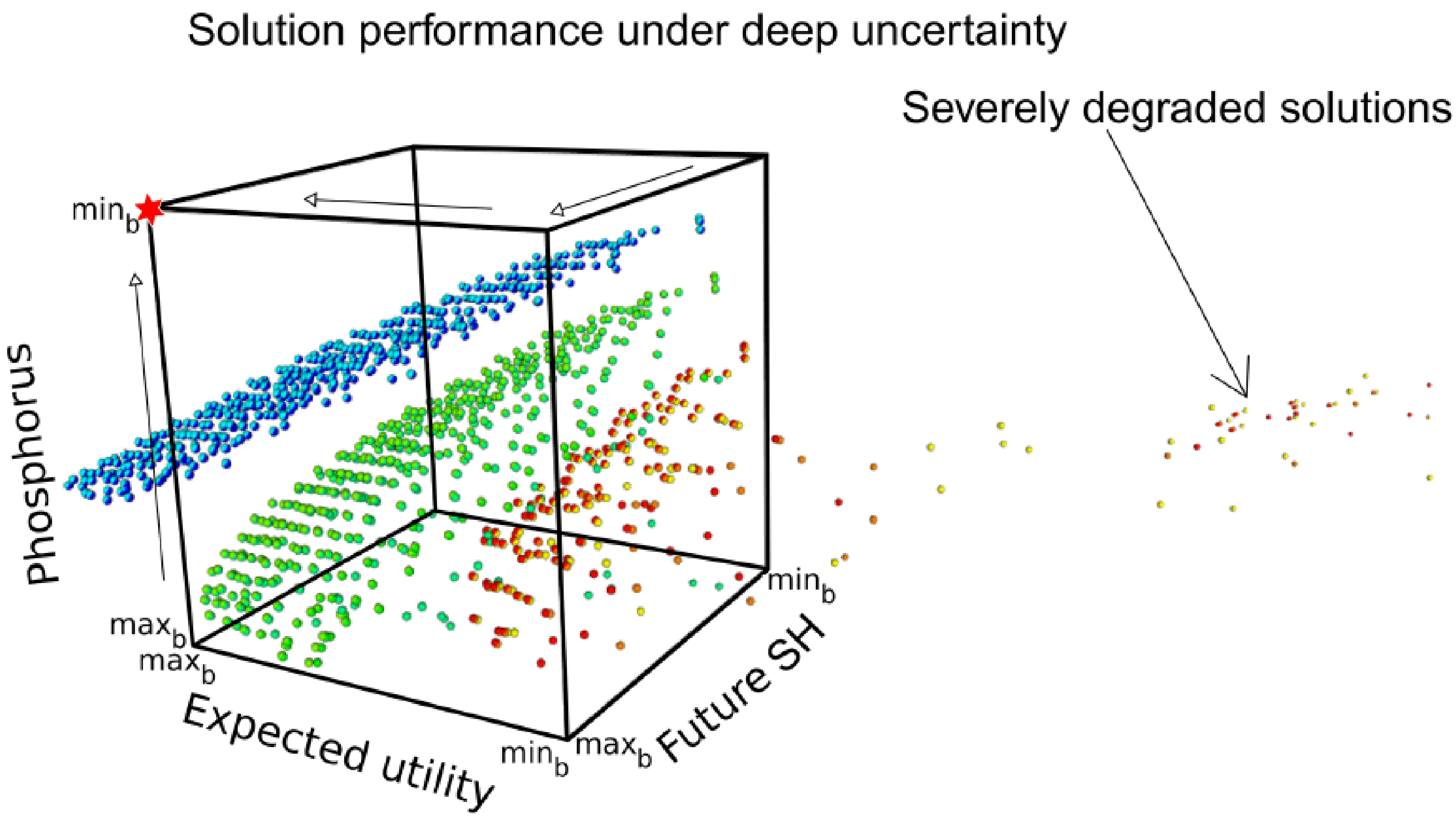
Selecting solution strategies:



Robustness for well-characterized uncertainties: satisfying performance thresholds across SOWs



Multi-objective regrets as assumptions change from baseline conditions



Robustness under deep uncertainty